

# Fermionic random transverse-field Ising spin chain

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The interplay of spin and charge fluctuations in the random transverse-field Ising spin chain on the fermionic space is investigated. The finite chemical potential, which controls the charge fluctuations, leads to the appearance of the quantum critical region in the phase diagram where the magnetic correlations are quenched by nonmagnetic sites. Regions of nonmonotonous temperature dependence of spin-spin correlation length appear at nonzero  $\mu$ . The results on the one-fermion density of states of the model are discussed.

## I. INTRODUCTION

The random transverse-field Ising spin chain (RTFISC) is the simplest nontrivial model exhibiting a quantum phase transition driven by quenched random interactions. The investigations of this model allowed to obtain many results which are of general importance for the quantum phase transitions in systems with random interactions<sup>1-4</sup>.

The extension of the RTFISC model on the fermionic space brings two aspects. From one hand, the original spin chain becomes a system of fermions with (statistical) charge fluctuations. That allows us to investigate the interplay of the charge and spin fluctuations at quantum phase transition.

Quantum phase transitions in fermionic systems with quenched disorder affect significantly the properties in both spin and charge sectors, which has been emphasized in a group of theoretical works<sup>5-10</sup> and suggested by experimental results for heavy fermion systems<sup>11,12</sup>. It will be shown in this paper that the statistical charge fluctuations affect the magnetic phase diagram of the chain, creating a quantum critical region close to the critical point. In that region the role of the charge fluctuations is analogous to the role of the thermal fluctuations at small finite temperature<sup>8,9</sup>. The fermionic description also represents a first step toward the introduction of electron hopping, which would allow us to investigate transport properties and a spatial structure of the electronic states<sup>13</sup>.

On the other hand, the extension of a spin model on the fermionic space provides the opportunity of a natural description of dilution of the magnetic model with nonmagnetic (empty and doubly occupied) sites. Working in a great canonical ensemble at a fixed chemical potential, one describes the annealed dilution.

Phase transitions in diluted magnetic systems have been a subject of numerous investigations (for a review, see Refs.<sup>14,15</sup>). In pure systems the annealed dilution leads to aggregation effects, whereas the separation of thermal and percolating contributions to correlation functions is proper to the quenched dilution<sup>14</sup>.

The RTFISC represents a case of the annealed site dilution in a system with quenched magnetic randomness. Due to the interplay between the quenched random magnetic interactions and the annealed dilution, the separation of magnetic and percolating contributions to the inverse spin-spin correlation length combines with the dependence of the dilution probability on the strength of magnetic fluctuations.

In this paper we analyze the random transverse-field Ising spin chain on the fermionic space (later referred to as fermionic chain), concentrating especially on the effects of a finite chemical potential  $\mu$  on the spin-spin correlation function and on the one-fermion density of states. The presented analysis is based on the real-space renormalization group (RG) method applied by D. Fisher in the paper<sup>1</sup>, further referred to as I.

It was shown in the work I, that the near-critical properties of the chain on the spin space (later referred to as spin chain) are described by the universal scaling distributions of the renormalized bonds and transverse fields. Our considerations show that the behavior of the fermionic chain is, however, nonuniversal and depends on the *original* distribution of transverse fields. The effect of bonds on the properties of the fermionic chain is described by the universal scaling distributions of the Ref. I.

The duality between the bonds and transverse fields, which is an important feature of the spin chain, is destroyed by the charge fluctuations in the fermionic chain. An interesting feature of the fermionic chain is the appearance of regions in the phase diagram at finite  $\mu$ , where the spin-spin correlation length changes nonmonotonously with temperature. In what follows, we provide an explanation for such quasireentrant behavior.

## II. GENERAL CONSIDERATION

In the great canonical ensemble, the Hamiltonian of the model under consideration reads

$$\hat{H} = \sum_i \left\{ -J_i \hat{S}_i^z \hat{S}_{i+1}^z - h_i \hat{S}_i^x - \mu \hat{n}_i \right\}, \quad (1)$$

where  $\hat{S}_i^\nu \equiv \hat{a}_{i,\alpha}^\dagger \sigma_{\alpha,\beta}^\nu \hat{a}_{i,\beta}$  is the operator of a  $\nu$ -component ( $\nu = x, z$ ) of the spin of the fermion at the site  $i$ ,  $\hat{n}_i = \hat{a}_{i,\alpha}^\dagger \hat{a}_{i,\alpha}$  is the particle number operator at the site  $i$ ,  $\hat{a}_{i,\alpha}^\dagger$  and  $\hat{a}_{i,\alpha}$  are the fermion operators representing a fermion on the site  $i$  with a  $z$ -component of the spin  $\alpha = \pm 1$ , and  $\sigma_{\alpha,\beta}^\nu$  denotes the element of the Pauli matrix.

Since there is no explicit charge dynamics (hopping) in the model, the statistical charge fluctuations (the nonmagnetic sites with occupation number 0 or 2) are determined by the distribution of the energy levels in the system. The distribution of the energy levels in turn follows from the distributions of random bonds and transverse magnetic fields. The strength of the charge fluctuations (the number of nonmagnetic sites) is controlled by the chemical potential and temperature.

At zero chemical potential and temperature, all sites in the chain are magnetic and the phase diagram of the fermionic chain is equivalent to that of the spin chain. The fermionic chain undergoes a quantum phase transition when the distributions of transverse magnetic fields  $h$  and ferromagnetic couplings  $J$  satisfy  $\delta = 0$  with  $\delta$  defined in I as  $\delta \equiv \frac{\ln h - \ln J}{\text{var}(\ln h) + \text{var}(\ln J)}$ .

At zero temperature, a nonmagnetic site breaks magnetic correlations between the spins on different sides of it. Due to the random space positions of the nonmagnetic sites, the whole chain is divided into magnetic fragments of random lengths. Therefore, the nonmagnetic sites introduce edge effects to spin-spin correlations in magnetic fragments.

Generally, the probability of a given configuration of nonmagnetic sites in the chain is a functional of the whole configuration. The situation simplifies, if the nonmagnetic sites occur very rarely in the chain, i.e. the distance between the neighbor nonmagnetic sites is large. Then, neglecting the edge effects of the nonmagnetic sites on the spin-spin correlations, one can approximately describe the distribution of nonmagnetic sites in terms of the probability of a single nonmagnetic site in the otherwise magnetic chain. We denote that probability as  $W(\mu)$ .

Given  $W$ , an important relation between the spin-spin correlation functions on the spin space and on the fermionic space can be derived. Since a nonmagnetic site breaks completely the correlation between spins on different sides of it, the spin-spin correlation function on the fermionic space  $\langle S_i^z S_{i+x}^z \rangle$  equals the correlation function on the spin space, if all the sites between  $i$  and  $i+x$  are magnetic, and is zero otherwise. For the mean spin-spin correlation functions, we obtain

$$C_f(x) = (1 - W)^x C_s(x), \quad (2)$$

where  $C_f(x)$  is the correlation function on the fermionic space,  $C_s(x)$  is the correlation function on the spin space, and  $(1 - W)^x$  expresses the probability that all the sites between  $i$  and  $i+x$  are magnetic. Note that the factorization of the correlations in Eq. (II), as well as the expression  $(1 - W)^x$  for the probability of a nonmagnetic fragment with length larger than  $x$ , is valid only in the limit of rare nonmagnetic sites. Using expression (II), we obtain the leading contribution to the spin-spin correlation function at small  $W$ .

At  $W \ll 1$  we can write, approximately,

$$C_f(x) = e^{-Wx} C_s(x), \quad (3)$$

then the influence of the nonmagnetic sites at finite  $\mu$  reduces to an additive contribution to the inverse correlation length. The separation of the inverse correlation length on the magnetic and nonmagnetic contribution is typical of one-dimensional systems with quenched dilution<sup>14</sup>.

Close to the critical point, the magnetic spin-spin correlation length diverges. Then, at finite  $\mu$ , the correlation length of the fermionic chain is determined by the value  $1/W(\mu)$ , which is the average distance between two neighbor nonmagnetic sites. By analogy with the effect of small temperature on the quantum phase transition<sup>8,9</sup>, we call the region, where  $W(\mu)$  dominates over the magnetic inverse correlation length, the quantum critical region. The relation between  $W(\mu)$  and the inverse correlation length of the spin chain determines the size of the quantum critical region.

In the quantum disordered phase the spin-spin correlation length can be represented in the form

$$\xi = \delta^{-2} \frac{1}{1 + W/\delta^2}, \quad (4)$$

where  $\delta^2$  is the inverse correlation length on the spin space found in I. From Eq. (4), one identifies formally the quantum fluctuations – percolation crossover exponent  $\phi = 2$  (Ref.<sup>14</sup>). At the same time, the probability  $W$  is a

function of  $\delta$ ,  $W = W(\mu, \delta)$ , and the relation between the chemical potential  $\mu$  and the distance from the critical point  $\delta$  at the crossover cannot be expressed in a power-law form.

In order to obtain the probability  $W(\mu)$ , we follow a given site during the applied in the paper I RG-process and find its ground state. To this end, consider an ensemble of chains with the transverse field at a given site  $P$  fixed to the value  $h_p$ . The probability of this ensemble is  $\rho(h_p)$ , where  $\rho(h)$  is the *original* distribution of the transverse fields. The next to the site  $P$  bonds,  $J_L$  and  $J_R$ , are random, and their distributions evolve in the RG process according to the RG-equations.

The main assumption of the RG procedure in I is that of broad near-critical distributions of renormalized bonds  $J$  and fields  $h$ . It follows from this assumption that the strongest transverse fields,  $h_i = \Omega = \max\{J_j, h_j\}$ , and the strongest bonds,  $J_i = \Omega$ , determine completely the ground states of the associated local parts of the chain, the couplings to the rest of the system being treated perturbatively. For the renormalization scale  $\Omega \gg \mu$ , the sites with couplings of the strength  $\Omega$  are singly occupied and the RG transformation is the same as in I. The spins at the sites with  $h_i$  in the infinitesimal interval  $d\Omega$  below  $\Omega$  are decimated to form effective bonds of the strength  $\tilde{J}_i = J_{i-1}J_i/\Omega$ , and the spins at the neighbor sites with the bond  $J_i$  in the interval  $[\Omega - d\Omega, \Omega]$  form spin clusters with effective transverse fields  $\tilde{h}_i = h_i h_{i+1}/\Omega$ . Then the RG scale  $\Omega$  is lowered infinitesimally by  $d\Omega$ .

At  $\mu \ll \Omega_I$ , where  $\Omega_I$  is the initial RG-scale, the typical length of magnetic fragments is large, and, neglecting the edge effects of the nonmagnetic sites, we assume the distributions of couplings in a magnetic fragment to be described by the RG-solutions of the paper I for the distributions of couplings in the infinite chain.

The ground state of the site  $P$  becomes defined in the moment when it is decimated in the RG process. The magnetic or nonmagnetic character of the ground state depends on the energy balance between the lowest magnetic and nonmagnetic states.

The site  $P$  can assume a nonmagnetic ground state in two ways.

(i) At a scale  $\Omega > h_p$ , the site is coupled (for example, from the right) with the strongest bond  $J_R = \Omega$  to a spin cluster. In this case, the small contribution of the other adjacent bond  $J_L$  to the energy of the magnetic state should be taken into account. The energy of the magnetic state is  $E_m = -\Omega - J_L - \mu$  and the energy of the nonmagnetic state is  $E_{nm} = -2\mu$ . The site becomes nonmagnetic if  $\mu - \Omega - J_{R,L} > 0$ .

(ii) The site is not decimated up to the scale  $\Omega = h_p$ . Then the transverse field  $h_p$  becomes the strongest coupling. The energy of the magnetic state is  $E_m = -\Omega - \mu$  and the energy of the nonmagnetic (doubly occupied) state is  $E_{nm} = -2\mu$ . Therefore, the ground state is nonmagnetic if  $\mu - \Omega > 0$ .

The ground state of a once decimated site does not change in the following RG transformations.

According to the assumption of rare nonmagnetic sites, we neglect the pairs of nonmagnetic sites and larger nonmagnetic clusters.

The site  $P$  remains undecimated to the scale  $\Omega$  if the *original* transverse field  $h_p$  is smaller than  $\Omega$  and none of the renormalized adjacent bonds  $\tilde{J}_{R,L}$  takes the strongest value from the beginning of the RG procedure up to  $\Omega$ .

On the logarithmic scale  $\Gamma = \ln(\Omega_I/\Omega)$ , we define the "survival" probability  $S(\zeta, \Gamma)$ .  $S(\zeta, \Gamma)$  is the probability that the effective bond  $\tilde{J}_R$ , adjacent from the right to the site  $P$ , never takes the strongest value (survives) up to the scale  $\Omega > h_p$  and equals  $\tilde{J}_R = \Omega \exp(-\zeta)$  at the scale  $\Omega$ . The function  $S(\zeta, \Gamma)$  satisfies the following RG-equation:

$$\frac{\partial S(\zeta, \Gamma)}{\partial \Gamma} = \frac{\partial S(\zeta, \Gamma)}{\partial \zeta} - R_0(\Gamma)S(\zeta, \Gamma) + R_0(\Gamma)S \otimes_{\zeta} P, \quad (5)$$

where  $R_0(\Gamma) = 2\delta/(1 - e^{-2\delta(\Gamma+C_0)})$  is the probability of the strongest transverse magnetic field at the logarithmic RG scale  $\Gamma = \ln(\Omega_I/\Omega)$ ,  $P(\zeta, \Gamma)$  is the probability density of the bond of the logarithmic strength  $\zeta = \ln(\Omega/J)$ , and  $C_0$  is a RG-irrelevant constant that can be obtained from the original distributions of couplings  $\pi(J)$  and  $\rho(h)$ . The functions  $R_0(\Gamma)$  and  $P(\zeta, \Gamma)$  were found in the work I. The explicit view of  $P(\zeta, \Gamma)$  is  $P(\zeta, \Gamma) = u(\Gamma)e^{-\zeta u(\Gamma)}$  with  $u(\Gamma) = 2\delta/(e^{2\delta(\Gamma+C_0)} - 1)$ .

The first two terms in Eq. (5) describe the reduction of the probability due to the decimation of the bonds of the strength  $\Omega$  and the overall rescaling of the energies in the RG transformation. The last two terms describe, respectively, the reduction of the bonds and the creation of effective bonds of the logarithmic strength  $\zeta$  by the decimation of the right neighbor site due to the strong transverse field. The last term is proportional to the convolution of the distributions of the bonds adjacent to the right neighbor site,  $S \otimes_{\zeta} P \equiv \int_0^{\zeta} d\zeta' S(\zeta', \Gamma)P(\zeta - \zeta', \Gamma)$ . Equation (5) looks

dual to Eq. (5.3) of paper I, which describes the survival probability of an end-point spin cluster.

The special solution of Eq. (5), satisfying the initial normalization  $\int_0^{\infty} d\zeta S(\zeta, \Gamma = 0) = 1$ , reads:

$$S(\zeta, \Gamma) = \delta e^{-C_0\delta} \frac{\sinh(C_0\delta)}{\sinh^2[(C_0 + \Gamma)\delta]} e^{-\zeta u(\Gamma)}. \quad (6)$$

The equation for the probability  $S(\zeta, \Gamma)$  and its solution are valid for the small values of the chemical potential  $\mu \ll \Omega_I$ . In this case the main contribution to the probability  $S(\zeta, \Gamma)$  comes from the late stages of the RG process, where the role of the RG-irrelevant nonuniversal parts of the distributions of effective bonds and transverse fields is minor.

The probability of the nonmagnetic ground state of the site with a fixed transverse field  $h_p$ ,  $W_h(\mu)$ , is given now as the sum of probabilities for two possible cases,  $W_h = w_J + w_h$ .  $w_J$  is the probability that the nonmagnetic site appears when one of the adjacent bonds, say  $J_R$ , takes the strongest value and the condition of the nonmagnetic ground state  $\Omega + J_L < \mu$  is fulfilled.  $w_h$  is the probability that none of the adjacent couplings becomes the strongest up to the scale  $\Omega = h_p$  and then, at  $h_p < \mu$ , the site assumes the nonmagnetic ground state.

Both probabilities,  $w_J$  and  $w_h$ , can be expressed through the "survival" probability  $S(\zeta, \Gamma)$  in the following manner:

$$w_J = 2 \int_{\Gamma_\mu}^{\Gamma_h} d\Gamma \int_{-\ln(\mu/\Omega_I - e^{-\Gamma})}^{\infty} d\xi S(0, \Gamma) S(\xi, \Gamma), \quad (7)$$

$$w_h = \left[ \int_0^\infty d\zeta S(\zeta, \Gamma_h) \right]^2 \vartheta(\mu - h) = \frac{e^{2\Gamma_h \delta} \sinh^2(C_0 \delta)}{\sinh^2[(C_0 + \Gamma_h) \delta]} \vartheta(\mu - h). \quad (8)$$

The prefactor 2 in Eq. (7) accounts for the fact that any of the two adjacent bonds can assume the strongest value.

The probability  $W_h$  is universal in the sense, that it depends on the scaling distribution of renormalized bonds and the details of the original distributions are irrelevant.

The whole probability  $W(\mu)$  obtains as an integral  $W(\mu) = \int_0^\mu dh \rho(h) W_h(\mu)$ , which is the average of the probability  $W_h(\mu)$  over the *original* distribution of transverse fields  $\rho(h)$ . The probability of the nonmagnetic site is therefore nonuniversal and depends of the details of the distribution of transverse fields  $\rho(h)$ . However, it contains a universal factor  $W_h(\mu)$ , which can be separated from the nonuniversal one.

### III. SPIN-SPIN CORRELATION FUNCTION AT ZERO TEMPERATURE.

Below we analyze the behavior of the function  $W_h(\mu)$  in different regions of the phase diagram and the behavior of the spin-spin correlation length  $\xi_f$  for two distributions of original transverse fields:

- a) the rectangular distribution of width  $b$ ,  $\rho(h) = (1/b)\vartheta(b - h)$  and
- b) the exponential distribution  $\rho(h) = (1/b)\exp(-h/b)$ .

As it was mentioned above, the inverse correlation length is the sum of the magnetic and the nonmagnetic contributions. The magnetic contribution is equal to that of the model on the spin space and the nonmagnetic one equals  $W$ . We consider different phases separately.

#### A. Deep in the quantum disordered phase, $\delta > 0$

In this region one can obtain the leading contribution for the probability density of the nonmagnetic ground state at  $h < \mu$  analytically, neglecting the influence of the weak neighbor bond in the calculation of  $w_J$ .  $W_h$  obtains in the form:

$$W_h \approx \sinh^2(C_0 \delta) \frac{e^{2\Gamma_\mu \delta}}{\sinh^2[(\Gamma_\mu + C_0) \delta]} \approx 4e^{-2C_0 \delta} \sinh^2(C_0 \delta) (1 - 2\tilde{\mu}^{2\delta} e^{-2C_0 \delta}), \quad (9)$$

where  $\Gamma_\mu \equiv -\ln(\tilde{\mu})$ ,  $\tilde{\mu} \equiv \mu/\Omega_I \ll 1$  is a reduced chemical potential.

The value  $\tilde{\mu}^{2\delta}$  orders the expansion of  $W_h$  at small  $\tilde{\mu}$ . The  $\mu$ -independent part of  $W_h$  is the probability that the site remains undecimated up to the scale  $\Omega = h$  in the chain on the spin space. The leading term of the probability  $W_h$  is  $h$ -independent, which leads to the factorization of the nonuniversal part of the probability of the nonmagnetic state  $W(\mu)$ ,

$$W(\mu) = W_h \int_0^\mu \rho(h) dh. \quad (10)$$

In the case of the rectangular distribution (a) with the width  $b > \mu$ , the spin-spin correlation length obtains

$$\xi_f \approx \left[ (\mu/b) \frac{e^{2\Gamma_\mu \delta} \sinh^2(C_0 \delta)}{\sinh^2[(\Gamma_\mu + C_0) \delta]} + \delta^2 \right]^{-1} \quad (11)$$

The first term in the square brackets is the nonmagnetic contribution and the second one is that obtained in the paper I contribution of the magnetic quantum fluctuations.

The correlation length for the distribution (b) obtains from Eq. (11) by the substitution of the prefactor  $(\mu/b) \rightarrow (1 - e^{-\mu/b})$ . The correlation length decreases with increase of  $\delta$ , i.e., deeper into the quantum disordered phase. The account for the weak couplings in calculation of  $w_J$  does not change this dependence.

### B. Ordered phase, $\delta < 0$

In the ordered phase the spontaneous magnetization is suppressed at finite chemical potential by the presence of the nonmagnetic sites. The correlation length equals  $\xi_f = 1/W(\mu)$ . The expressions for  $W_h$  and  $W$ , obtained for the disordered phase, are valid here if considered at  $\delta < 0$ . The leading behavior of the function  $W_h(\mu)$ , and hence of the correlation length at small  $\tilde{\mu}$  is, however, different. At  $\tilde{\mu} \ll 1$  we obtain

$$W_h \approx 4\tilde{\mu}^{4|\delta|} e^{-2C_0\delta}. \quad (12)$$

The function  $W(\mu)$  differs from  $W_h(\mu)$  by the prefactor  $\int_0^\mu \rho(h)dh$ , which was calculated for the two distributions  $\rho(h)$  in the previous subsection. Now  $W_h$ , and hence  $W$ , decreases with  $|\delta|$ , i.e., the correlation length  $1/W$  increases deeper into the ordered phase.

### C. Close to the critical point. Quantum critical region.

Close to the critical point, for  $\tilde{\mu} \gg e^{-1/\delta}$ , the expansion of  $W_h(\mu)$  in powers of  $\mu^{2\delta}$  is not valid anymore, and one should make a double expansion in  $1/|\ln \tilde{\mu}|$  and  $\delta$ . The leading term of  $W_h(\mu)$  close to the critical point reads

$$W_h \propto \frac{C_0^2}{(C_0 - \ln \tilde{\mu})^2}. \quad (13)$$

The leading term of  $W_h$ , and hence of the nonmagnetic contribution to the inverse correlation length, is  $\delta$ -independent. It is determined by the chemical potential alone, and this regime can be associated with the quantum critical (QC) regime of the fermionic chain. This definition of the QC regime reflects the change of the structure of the small- $\tilde{\mu}$  expansion. It results in the crossover line  $\tilde{\mu} \sim e^{-1/(2\delta)}$  between the QC and quantum disordered regimes. From the other side, the more physical definition of the QC region is as a region, where the nonmagnetic contribution to the inverse correlation length is larger than the magnetic one.

The crossover curves, which correspond to the discussed above definitions, are shown in Fig. 1. As one crosses the dashed curve in increasing  $\tilde{\mu}$ , the leading nonmagnetic contribution to the inverse correlation length becomes  $\delta$ -independent. Below the solid line the magnetic contribution dominates (quantum disordered regime), whereas above it the correlation length is determined by the nonmagnetic contribution.

The form of the distribution  $\rho(h)$  and the constant  $C_0$  correct the position of the solid line, the qualitative behavior being unchanged. Rising  $\mu$ , there are different scenarios of the crossover from the quantum disordered to the quantum critical regime depending upon which, solid or dashed, line is intersected first. If the dashed line is intersected first, then one goes from the disordered phase into the region where the leading nonmagnetic contribution is already  $\delta$ -independent, but still smaller than the magnetic one. Further, as the solid line is intersected, one enters the region of the dominance of the nonmagnetic contribution. This scenario takes place at comparatively large and at very small  $\delta$ .

If the solid line is intersected first, then one enters the region where the nonmagnetic contribution to the correlation length dominates, still being  $\delta$ -dependent. After that, the leading contribution to the correlation function becomes  $\delta$ -independent.

## IV. SPIN-SPIN CORRELATION FUNCTION AT SMALL NONZERO TEMPERATURE.

The similarity of finite  $\mu$  and finite  $T$  effects on the quantum phase transition inspires us to investigate the question of the interplay of these two factors. Thermal fluctuations result in the nonzero occupation of the excited states. The fluctuations from the nonmagnetic ground state to the magnetic excited state restore, partially, the correlations between spins of different magnetic fragments. At the same time, the finite temperature destroys the correlations

between spins of the same magnetic fragment. The competition between these two effects determines the physics of the model at small finite temperature.

Here the regime  $T \ll \mu \ll \Omega_I$  is considered. Then only the first excited state may be taken into account. The consideration changes in only one point in comparison with the zero-temperature case. Namely, in defining the probability of a nonmagnetic site, the condition of the nonmagnetic ground state is replaced by the probability that the site is in the nonmagnetic state, whether it is the ground or the excited state. The probability that the site is in the nonmagnetic state can be written as

$$p(\Omega, \mu, T) = 1/(1 + \exp[(\Delta - \mu)/T]). \quad (14)$$

Here  $\Delta - \mu$  denotes the energy gap between the ground and the first excited state.  $\Delta = \Omega$  if  $\Omega$  is a transverse field,  $\Delta = \Omega + J_{R,L}$  if  $\Omega$  is a bond, and  $J_{R,L} < \Omega$  is the other bond adjacent to the site. The finite temperature therefore causes the smearing of the probability of the nonmagnetic state  $\vartheta(\mu - \Omega)$  over the region of order  $T$  around  $\mu = \Omega$ .

The formulas for the probabilities  $w_J$  and  $w_h$ , Eqs. (7) and (8), become

$$w_J = 2 \int_0^{\Gamma_h} d\Gamma \int_{-\ln(\mu/\Omega_I - e^{-\Gamma})}^{\infty} d\xi S(0, \Gamma) S(\xi, \Gamma) / (1 + \exp[(e^{-\Gamma} + e^{-\xi} - \tilde{\mu})/\tilde{T}]), \quad (15)$$

$$w_h = \left[ \int_0^{\infty} d\zeta S(\zeta, \Gamma_h) \right]^2 (1 + \exp[(h - \mu)/T])^{-1}, \quad (16)$$

where  $\tilde{T} \equiv T/\Omega_I$ .

The temperature behavior of the correlation length was analyzed numerically. The dependence of  $W_h(T)$  on temperature is monotonously decreasing at small transverse field  $h$ , but it changes to the nonmonotonous at larger fields  $h \sim \mu$ . The resulting temperature dependence of the probability of the nonmagnetic state  $W(T)$  depends strongly upon the original distribution of transverse fields and can also be nonmonotonous.

The effect of breaking of the magnetic correlations by thermal fluctuations is particularly strong in the quantum disordered phase and at the critical point, where it contributes to the inverse correlation length as  $\pi^2/(\ln \tilde{T})^2$  and  $\pi^2/(2 \ln \tilde{T})^2$ , respectively, (see I). This effect dominates in the quantum disordered phase and at the critical point, and leads to a monotonously decreasing overall temperature dependence of the correlation length.

In the ordered phase, however, the temperature breaking of the correlations is weaker and contributes to the inverse correlation length as  $4\delta^2(\tilde{T})^{2|\delta|}$  (Ref.I). In this phase the temperature dependence of the inverse correlation length is nonmonotonous if the distance from the critical point  $\delta$  exceeds some finite value (see Fig. 2).

## V. DEPENDENCE OF AVERAGED FILLING ON CHEMICAL POTENTIAL $\nu(\mu)$ . ONE-FERMIONIC DENSITY OF STATES.

The obtained expressions for the probability of the nonmagnetic site allow us to calculate the dependence of the average filling on  $\mu$ . For  $\mu > 0, T = 0$ , each nonmagnetic site is doubly occupied and the average filling can be written as

$$\bar{\nu}(\mu) = \lim_{l \rightarrow \infty} (1/l) \sum_{i=1}^l (1 - W + 2W) = 1 + W(\mu), \quad (17)$$

where  $W$  is the probability of a nonmagnetic site ( $\nu = 2$ ) and  $1 - W$  is the probability of a magnetic site ( $\nu = 1$ ). Substituting the expressions for  $W(\mu)$ , we obtain for the average filling the following formula:

$$\bar{\nu} \approx 1 + B(\mu, b) \frac{e^{2\Gamma_\mu \delta} \sinh^2(C_0 \delta)}{\sinh^2[(C_0 + \Gamma_\mu) \delta]}, \quad (18)$$

where  $B(\mu, b) = \int_0^\mu \rho(h) dh$  is a specific for the original distribution of the transverse fields factor,

$$B(\mu, b) = \begin{cases} \mu/b & \text{for the distribution (a)} \\ 1 - e^{-\mu/b} & \text{for the distribution (b).} \end{cases} \quad (19)$$

Equation (18) describes the average filling in the quantum disordered phase at  $\delta > 0$  and in the ordered phase at  $\delta < 0$ . In the limit  $\delta \rightarrow 0$ , one obtains  $\bar{\nu}(\mu)$  at the critical point as

$$\bar{\nu} \approx B(\mu, b) \frac{C_0^2}{(C_0 - \ln \tilde{\mu})^2}. \quad (20)$$

Taking the derivative  $d\bar{\nu}/d\mu$ , one can analyze the behavior of the average density of states. Close to the half-filling ( $\mu \ll 1$ ), one obtains the average density of states in the form

$$\frac{d\bar{\nu}}{d\mu} = (1/b) \frac{\tilde{\mu}^{-2\delta} \sinh^2(C_0\delta)}{\sinh^2[(C_0 - \ln \tilde{\mu})\delta]} \left( 1 + \frac{2\delta \tilde{\mu}^\delta e^{-C_0\delta}}{\sinh[(C_0 - \ln \tilde{\mu})\delta]} \right) \quad (21)$$

for both distributions (a) and (b) of transverse fields.

In the disordered phase ( $\delta > 0$ ), the density of states at half filling is finite and equals

$$\frac{d\bar{\nu}}{d\mu} = (4/b) e^{-2C_0\delta} \sinh^2(C_0\delta). \quad (22)$$

The behavior of the density of states in the quantum disordered phase is highly nonuniversal and depends strongly on the original distribution of transverse fields. For example, the average density of states would acquire a gap if there were a gap in the original distribution of transverse fields, i.e., a nonzero minimal value of the original transverse field.

In the ordered phase ( $\delta < 0$ ), the behavior of the density of states is more universal in the sense that there is a gap in the density of states at half filling if the function  $B(\mu)$  grows more slowly than  $\mu^{-4|\delta|}$  as  $\mu$  goes to zero. The density of states for the distributions (a) and (b) reads

$$\frac{d\bar{\nu}}{d\mu} = \frac{4}{b} \sinh^2(C_0\delta) e^{-2C_0|\delta|} (1 + 4|\delta|) \tilde{\mu}^{4|\delta|}. \quad (23)$$

The soft gap behavior of the average density of states at half filling survives at the critical point as well. Here the condition for the function  $B(\mu)$  is that it should grow more slowly than  $(\ln \mu)^2$  as  $\mu$  goes to zero. The expression for the average density of states for cases (a) and (b) at the critical point looks like

$$\frac{d\bar{\nu}}{d\mu} = (1/b) \frac{C_0^2}{(C_0 - \ln \tilde{\mu})^2} \left( 1 + \frac{2}{C_0 - \ln \mu} \right). \quad (24)$$

## VI. CONCLUSION

We showed that the extension of the RTFISC model on the fermionic space, introducing statistical charge fluctuations, brings about interesting features in the magnetic as well as in the fermionic characteristics. Strong mutual effects of magnetic and charge fluctuations show up in the power-law scaling of the contribution of charge fluctuations to the inverse spin-spin correlation length with the distance from the magnetic quantum critical point and also in the appearance of the quantum critical region in the phase diagram.

Although the characteristics in the charge sector (percolating correlation length, fermion density of states) behave nonuniversally, they contain a universal factor, which can be separated from the nonuniversal one. The nonuniversal features enter through the original distributions of transverse fields, whereas the bonds are described by universal scaling distributions. Then, it is natural that the nonuniversal behavior is particularly strong in the quantum disordered phase, where the transverse fields dominate.

The results on the one-fermion density of states suggest different structures of electronic states in the ordered and in the quantum disordered phases in the model with electron hopping. The investigation of this model is left for the future.

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FIG. 1. Crossover lines  $\tilde{\mu}(\delta)$  between different regimes in the phase diagram calculated for the distribution (b) with the width  $b = 1/2$ . Above the solid line the nonmagnetic contribution to the inverse correlation length dominates. Above the dashed line the leading nonmagnetic contribution is  $\delta$ -independent.  $C_0 = 1$ .

FIG. 2. The temperature dependence of the correlation length in the ordered phase becomes nonmonotonous as the distance  $|\delta|$  exceeds some value.  $\xi(T)$  for the distribution (b) with  $b = 1$ .  $C_0 = 1$ ,  $\delta = -0.4$ ,  $\tilde{\mu} = 0.3$ .